Scattering on the lateral one-dimensional superlattice with spin-orbit coupling

D.V. Khomitsky[*]

Department of Physics, University of Nizhny Novgorod,
23 Gagarin Avenue, 603950 Nizhny Novgorod, Russian Federation
(Dated: February 6, 2008)

The problem of scattering of the two-dimensional electron gas on the lateral one-dimensional superlattice both having different strengths of Rashba spin-orbit coupling is investigated. The scattering is considered for all the electron states on a given Fermi level. The distribution of spin density components along the superlattice is studied for the transmitted states where the formation of standing waves is observed. It is found that the shape of spin density distribution is robust against the variations of the Rashba coupling constants and the Fermi level in the electron gas.

I. INTRODUCTION

In two-dimensional semiconductor heterostructures the spin-orbit (SO) interaction is usually dominated by the Rashba coupling[1] coming from the structure inversion asymmetry of confining potential and effective mass difference. The interest to these structures is related to the possible effects in charge and spin transport which produce novel ideas on the spin control in semiconductor structures and give rise to the applications of spintronics.[2] The idea to control the spin orientation in the beam of particles by means of SO coupling has been proposed in terms of spin optics.[3,4] In particular, the scattering on the border of two half-spaces each having a different value of SO coupling constants was studied.[3] It was shown that the spin orientation in transmitted wave strongly depends on the chirality of the incident one as well as on the angle of incidence and the angles of total reflection exist. Later the same authors applied their results for the case of spin polarizing in a system consisted of ballistic and diffusive regions.[4] One of the possible ways to control the band and the spin structure is to apply the gated structures with externally tuned periodic electric potential. In our recent paper we studied quantum states and the electron spin distribution in a system combining the spin-splitting phenomena caused by the SO interaction and the external periodic electric potential.[5] In the present paper me make an extensive use of these results for investigation of the problem of scattering for 2DEG with Rashba SO coupling on the SO superlattice. We solve the scattering problem on the SO superlattice occupying a half-space and study the transmitted states as a function of the Fermi energy of the incoming states. For the transmitted states the space distribution of spin density components is calculated for different values of Rashba coupling on both sides of the interface, for various amplitudes of the Fermi level position in the 2DEG.

The paper is organized as follows. In Sec.II we formulate the scattering problem and describe the incoming, reflected, and transmitted states. We also briefly discuss the structure of the eigenstates of the SO superlattice. In Sec.III the space distribution of spin density in the trans-
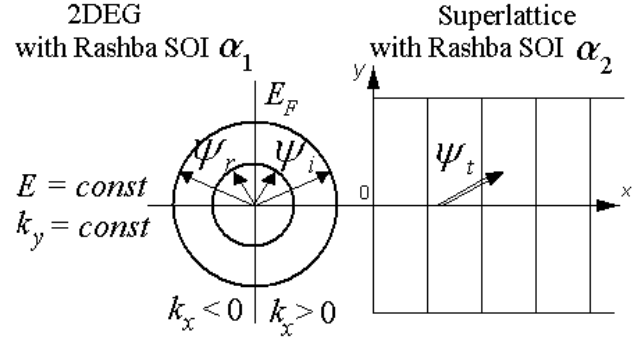

FIG. 1: Geometry of scattering of 2DEG with Rashba SO interaction on the spin-orbit lateral superlattice. The incoming ψ_i and reflected ψ_r spinors are the eigenstates of Rashba Hamiltonian with spin-orbit coupling constant α_1 and wavevectors belonging to the same Fermi contour. The transmitted states ψ_t are the Bloch spinors corresponding to another spin-orbit coupling constant α_2.

mitted state is calculated, and different cases of Rashba coupling on both sides of the interface are discussed. The concluding remarks are given in Sec.IV.

II. THE SCATTERING PROBLEM

We consider the scattering of electrons with spin-orbit coupling constant α_1 on the one-dimensional superlattice occupying a half-space $x > 0$ and also having a spin-orbit Rashba term with another value of Rashba coupling constant α_2. The incoming and reflected spinors are the eigenstates of Rashba Hamiltonian and belong to the same Fermi energy of the 2D electron gas. The transmitted states are the Bloch spinors with each of the components possessing the Bloch theorem. In addition to the energy, the k_y component of the momentum is conserved since the system is homogeneous in the y direction, as it is shown schematically in Fig.1.

The half-space $x < 0$ is the semiconductor structure with 2DEG characterized by the effective mass m and Rashba spin orbit coupling strength α_1. The quantum

states here are the eigenstates of the Rashba Hamiltonian $\hat{H}_0 = \hat{p}^2/2m + \alpha_1(\hat{\sigma}_x\hat{p}_y - \hat{\sigma}_y\hat{p}_x)$ where $\hbar = 1$. The eigenstates of this Hamiltonian are two-component spinors $\psi_i = \psi_{\mathbf{k}\lambda} = e^{i\mathbf{k}\mathbf{r}}(1, \quad e^{i\theta})/\sqrt{2}$ where $\lambda = \pm 1$ and $\theta = \arg[k_y - ik_x]$. The energy of the state is $E_0(\mathbf{k}, \lambda) = \frac{k^2}{2m} + \lambda\alpha_1 k$. It should be stressed that this wavefunction does not exhibit any spin texture $S_i = \psi^\dagger\hat{\sigma}_i\psi$, i.e. it determines a uniform space distribution of all spin density components $S_x = \lambda\cos\theta_0$, $S_y = \lambda\sin\theta_0$, and $S_z \equiv 0$. The idea of the system setup in Fig.1 is to convert this uniform distribution into a non-trivial spin texture by using a superlattice.

The incoming state is scattered on the border of the SO superlattice occupying the area at $x > 0$. In the left part of the space $x < 0$ there is the reflected state which is the linear combination of all eigenstates of Rashba Hamiltonian with the same energy as the incoming state and with $k_x < 0$. The wavevector modules are equal to $k_{1,2} = \sqrt{2mE + (m\alpha)^2} \pm m\alpha_1$, and the k_x component for each $k_{1,2}$ at fixed k_y is given by the usual relation $k_{1,2x} = \sqrt{k_{1,2}^2 - k_y^2}$. Thus, the reflected state at $x < 0$ has the following form:

$$x < 0: \quad \psi_{\mathrm{r}} = r_1 \frac{e^{-ik_1 x + ik_y y}}{\sqrt{2}} \begin{pmatrix} 1 \\ -e^{i\theta_1} \end{pmatrix} +$$
$$+ r_2 \frac{e^{-ik_2 x + ik_y y}}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\theta_2} \end{pmatrix}. \qquad (1)$$

Here the phases are defined by the momentum components as $\theta_{1,2} = \arg[k_y - ik_{1,2x}]$ and $r_{1,2}$ are the reflection coefficients which will be found below.

On the right-hand side in Fig.1 at $x > 0$ the transmitted electrons travel trough the SO superlattice. The transmitted state is the linear combination of the eigenstates of the SO superlattice with the energy and k_y equal to those of the incoming state:

$$x > 0: \quad \psi_{\mathrm{t}} = \sum_j c_j \psi(k_j, k_y) \qquad (2)$$

where the coefficients c_j can be found from the boundary conditions. The wavefunctions $\psi(k_j, k_y)$ are the Bloch eigenstates of the Hamiltonian in the SO superlattice having the form[5]

$$\psi_{s\mathbf{k}} = \sum_{\lambda n} a_{\lambda n}^s(\mathbf{k}) \frac{e^{i\mathbf{k}_n\mathbf{r}}}{\sqrt{2}} \begin{pmatrix} 1 \\ \lambda e^{i\theta_n} \end{pmatrix}, \quad \lambda = \pm 1 \quad (3)$$

where k_x is the quasimomentum in the 1D Brillouin zone $-\pi/a \le k_x \le \pi/a$, s is the band number, and $\theta_n = \arg[k_y - ik_{nx}]$. The coefficients $a_{\lambda n}^s$ are found by diagonalization of the superlattice Hamiltonian in the basis of Rashba spinors. The 1D superlattice potential in our problem can be chosen in the simplest form

$V(x) = V_0\cos(2\pi x/a)$ where a is the superlattice period and V_0 is the potential strength.

The scattering on the interface at $x = 0$ is described by the boundary conditions. For the problem considered in the paper these conditions have the form of the continuity equations which follow from the Schrödinger equation and can be written as

$$\psi\,|_{x=0-} = \psi\,|_{x=0+}, \qquad (4)$$
$$\hat{v}_x\psi\,|_{x=0-} = \hat{v}_x\psi\,|_{x=0+} \qquad (5)$$

where the velocity operator

$$\hat{v}_x = \frac{\partial\hat{H}}{\partial k_x} = \frac{\hat{p}_x}{m} - \alpha\hat{\sigma}_y. \qquad (6)$$

The equations (5) link the wavefunction $\psi_{\mathrm{i}} + \psi_{\mathrm{r}}$ at the left half-space $x < 0$ and the wavefunction ψ_{t} at the right half-space $x > 0$. Since both of the equations in (5) are written for two-component spinors, one has a system of four algebraic inhomogeneous equations describing the scattering which can be easily solved.

The quantum numbers which remain to be good during the scattering on 1D superlattice are the k_y component of the momentum and the energy of the incoming state. Here one has to distinguish the case when the energy of the incoming state at fixed k_y is within the limits of one of the superlattice bands and when this energy corresponds to a gap in the superlattice spectrum. The first case corresponds to the solution of system (5). For the second case the solution to the Schrödinger equation is not finite on the whole x axis and thus there are no states which propagate from the scattering interface through the superlattice. We call such case as a case of total reflection in analogy with optical scattering. It should be mentioned that such effect was already observed for the scattering of the Rashba states on the interface between two areas with different SO constant.[3] The states which do not propagate through the superlattice and are localized at the interface border are known as Tamm states. Such states were studied previously both in bulk crystals[7,8] and later in the superlattices.[9,10,11] In the latter case it was shown that typically the Tamm states decay inside the superlattice on the length of several periods with different results varying from two - three[9] to five - seven[11] lattice periods. In our case these results mean that the typical penetration length of Tamm states will be of the order of 100 - 700 nm which is substantially smaller than the total length of superlattices actually used in the present experiments. Hence, there will be no detection of such states with the possible device mounted after the superlattice. Thus, we neglect the Tamm states localized at the interface and consider only the Bloch states with the energy belonging to the bands of the superlattice which were discussed above.

III.   SPIN TEXTURE OF THE TRANSMITTED STATE

When the transmitted state (2) is fully determined, one can calculate the space distribution of the spin density $\psi^\dagger \hat{\sigma}_i \psi$ for the transmitted state which depends on the wavevector and polarization of the incident state. In a real experimental setup of 2DEG structure the electrons occupy not a single state with a given wavevector and polarization but all of the states on the Fermi level, as it is shown schematically in Fig.1. The electrons with $k_x > 0$ travel to the scattering interface and take part in the scattering process. Thus, it is reasonable to calculate the spin density for all the electrons with a chosen Fermi energy and $k_x > 0$ giving us the spin density distribution which can be actually probed by a detector,

$$S_i(x,y) = \int_{k_{Fx}>0} \psi_{\mathrm{t}}^\dagger \hat{\sigma}_i \psi_{\mathrm{t}} dk. \qquad (7)$$

Since the system is homogeneous in the y direction, one may consider only the x-dependence of (7) which may show some non-trivial spin texture along the superlattice. As it was mentioned above, the incident state of 2DEG with Rashba spin-orbit coupling has a space-independent spin density distribution. Below we shall see that a non-uniform spin density distribution which can be actually probed by a detector may be created by scattering on the SO superlattice.

First, let us consider a case when the Rashba coupling constant α_1 in the 2DEG on the left is substantially smaller than the parameter α_2 in the superlattice. This situation corresponds, for example, to the GaAs-based structure attached to the InAs-based SO superlattice. The results for the spin density distribution along the superlattice for $\alpha_1 = 0.1\alpha_2$ are shown in Fig.2 for the amplitude of the periodic potential $V_0 = 5$ meV and for the values of the Fermi energy $E_F = 10$ meV and $E_F = 30$ meV of the incident state. The upper plot on each figure shows the (S_x, S_z) projections of the spin density (7) while the lower one demonstrates the space dependence of (S_x, S_y) components. The space distance on the plot is measured in units of superlattice period $a = 60$ nm and starts at $n \gg 1$ which means that the spin detector is located far away from the superlattice border. The spin texture in Fig.2 has several remarkable features. First of all, it has a non-zero component S_z which is absent in spin density of the uniform 2DEG with Rashba SO coupling. As for the spin expectation values $\sigma_i = \int S_i dx$ for our problem, one has in general $\sigma_x = \sigma_z = 0$ and $\sigma_y \neq 0$ which follows from the symmetry considerations of the system (see Fig.1). Indeed, the system is symmetrical with respect to y sign reversal which means for Rashba SO coupling that $\sigma_x = 0$. The Rashba SO interaction also can not create the z polarization of 2DEG and thus $\sigma_z = 0$, as in the initial state. It should be noted that a similar feature was observed previously for the eigenstates in the SO superlattices at given quantum numbers
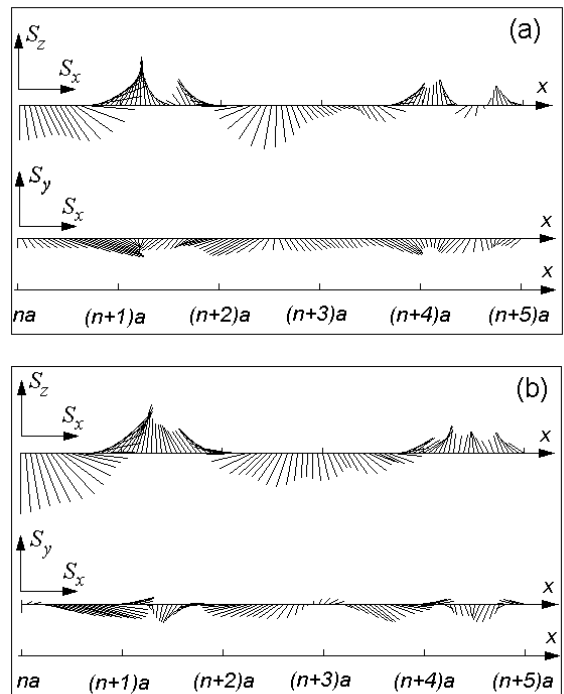

FIG. 2: Spin texture along the superlattice for the Rashba constant $\alpha_2 = 3 \cdot 10^{-11}$ eVm inside and $\alpha_1 = 0.1\alpha_2$ outside the superlattice. The periodic potential amplitude $V_0 = 5$ meV and the Fermi energy is (a) $E_F = 10$ meV and (b) $E_F = 30$ meV.

(k_x, k_y) in the Brillouin zone.[5] The only symmetry breaking caused by the scattering interface cancels the x sign reversal symmetry, making only the states with $k_x > 0$ to be actually scattered. Thus, one can see in Fig.2 and below in Fig.3 that one sign of $S_y(x)$ dominates, leading in those cases to a nonzero expectation value σ_y. The other reason is that the contributions to the spin expectation value σ_y from two parts of Fermi contours of the Rashba bands with $\lambda = \pm 1$ (see Fig.1) do not compensate each other due to the distance $2m\alpha_1$ between the Fermi radii. Another interesting feature of the spin density distribution in Fig.2 is that it does not repeat itself on the distance of one superlattice period. The explanation is that the transmitted state (2) consists of the Bloch spinors with different k_x components of the quasi-momentum providing the different partial wavelengths. As one can see from Fig.2, the approximate space period for the spin density is about several superlattice periods and, as our calculations have shown, does not depend on particular starting point $x = na$ if the condition $n \gg 1$ is satisfied. The latter means that the spin density detector is located far away from the scattering border, as it is supposed to be in real experiments. This circumstance allows to neglect the influence of the second right-hand border of the superlattice while solving the scattering problem.

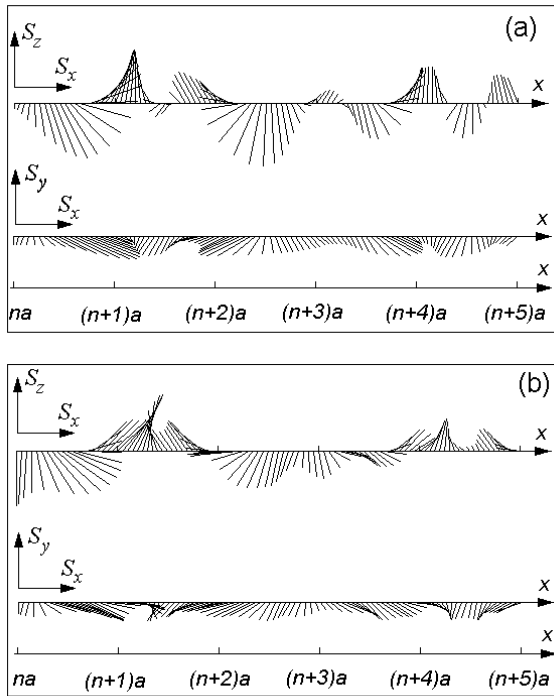Now we turn our attention to the opposite case $\alpha_2 =$

FIG. 3: Spin texture along the superlattice for the Rashba constant $\alpha_1 = 3 \cdot 10^{-11}$ eVm outside and $\alpha_2 = 0.1\alpha_2$ inside the superlattice. The periodic potential amplitude $V_0 = 5$ meV and the Fermi energy is (a) $E_F = 10$ meV and (b) $E_F = 30$ meV.

$0.1\alpha_1$ which can be realized experimentally, for example, by the GaAs-based SO superlattice attached to the InAs-based 2DEG. The results for the spin density distributions are presented in Fig.3. Again one can see the similarity between all the spin density textures in Fig.3 and Fig.2. The integral spin density distribution (7) maintains qualitatively the same form for different values of system parameters since it is sensible only to the global characteristics of the energy spectrum of the superlattice

which remain unchanged under variation of the Fermi level position and Rashba coupling strength. We have also observed that the results presented above are qualitatively the same for different values of the superlattice potential. Such robust spin density shape indicates that the effects discussed in the paper should survive under various perturbations which were left out of the scope in the present work such as defects and finite temperature. This conclusion can be justified further if we mention that the energy scale of the problem studied above belongs to the interval of $10\ldots30$ meV, which means that the effects discussed in the paper should be clearly observable at helium, and possibly also at nitrogen temperatures.

IV. CONCLUSIONS

We have studied the scattering of two-dimensional electron gas on the one-dimensional superlattice where the spin-orbit coupling was taken into account for both systems. The space distribution of spin density components was calculated for different values of Rashba coupling on both sides of the interface and for various Fermi level position. The observed shape of spin density standing waves is found to be insensitive to particular values of the electron Fermi energy and Rashba coupling strength indicating that the effects discussed in the paper should survive under various perturbations such as defects and finite temperature. The scale of energy involved in the processes discussed in the paper makes the results to be promising for experimental observation.

* Electronic address: khomitsky@phys.unn.ru
1 E.I. Rashba, Fiz. Tverd. Tela (Leningrad) **2**, 1224 (1960) [Sov. Phys. Solid State **2**, 1109 (1960)]; Y.A. Bychkov and E.I. Rashba, J. Phys. C **17**, 6039 (1984).
2 I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
3 M. Khodas, A. Shekhter, and A.M. Finkel'stein, Phys. Rev. Lett. **92**, 086602 (2004).
4 A. Shekhter, M. Khodas, and A.M. Finkel'stein, Phys. Rev. B **71**, 125114 (2005).
5 V.Ya. Demikhovskii and D.V. Khomitsky, JETP Letters **83**, iss.8, p.340 (2006) [Pis'ma v ZhETF **83**, iss.8, p.399

(2006)].
6 D. Grundler, Phys. Rev. Lett. **84**, 6074 (2000).
7 E.T. Goodwin, Proc. Cambridge Philos. Soc. **35**, 221 (1939).
8 S.D. Kevan, N.G. Stoffel, and N.V. Smith, Phys. Rev. B **32**, 4956 (1985).
9 H. Ohno, E.E. Mendez, J.A. Brum, J.M. Hong, F. Agulló-Rueda, L.L. Chang, and L. Esaki, Phys. Rev. Lett. **64**, 2555 (1990).
10 H.K. Sy and T.C. Chua, Phys. Rev. B **48**, 7930 (1993).
11 J. Kłos and H. Puszkarski, Phys. Rev. B **68**, 045316 (2003).